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Finite Element Analysis of Fatigue Life Assessment of Butt & Fillet Welded Joint with Cracks

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The S-N curves based fatigue designs of welded components and structures do not fully represent the weld details like effects of geometry, welding process and material defects. Experimental fatigue life prediction of welded joints is time consuming and costly. These limitations have prompted the need for adopting other ways for modeling the fatigue process rather than simply relating applied stress and fatigue life as in the S-N curves. A recent trend in this regard is the use of local approaches like fracture mechanics based attempts to model the whole fatigue process by considering the above effects. The stress at the crack tip can be accurately evaluated by means of linear elastic fracture mechanics by using suitable singular crack tip elements and proper mesh density. This can be further used to predict the fatigue life of a welded joint by using Paris's law. The final crack length of a welded joint can be found out by virtual crack extension method (VCEM), where the final crack length is that crack length at which for a given loading the stress intensity factor of a crack approaches or exceeds an upper limit known as critical stress intensity factor.

Keywords: Crack tip stress method, Critical stress intensity factor, LEFM, Plastic zone size, Strain energy release rate (SERR), Stress intensity factor (SIF), Virtual Crack Extension Method(VCEM)

1. INTRODUCTION:

Cracks in welded structures occur due to hydrogen embrittlement, improper fusion of base metal to weld metal and residual stresses. Experiments and analytical works indicate that the plastic deformation at the crack edges is very limited. Fatigue design of welded components and structures which are normally based on S-N curves do not represent the other details like weld geometry, welding process and material properties[1]. It is known that fatigue failures in structural members originate from extremely localized phenomena. Therefore, fatigue assessments should be based on the local parameters of geometry, loading and material.

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It is also important to consider the factors like welding residual stresses, welding distortions and inhomogeneous material in the critical area while predicting the fatigue life of a welded joint. Fatigue strength and service life of structural members with an existing crack can be determined according to the crack propagation approach[2]. The crack propagation rate of a crack subject to cyclic loading can be analyzed based on the cyclic stress intensity factor on the basis of a Paris law. Crack propagation occurs as soon as the threshold value of stress intensity factor in exceeded the critical stress intensity factor[3]. The presence of plasticity at the crack tip causes concern about the singularity of the stress intensity factor 'K_I'. This led to several investigations to understand the effect of plastic zone on estimation of linear elastic fracture mechanics (LEFM) criterion[4]. The stress at the crack tip can be accurately evaluated by means of linear elastic fracture mechanics by using suitable singular crack tip elements and mesh density [5]. If stress intensity of a crack approaches or exceeds an upper limit known as critical stress intensity factor (which is a material property) usually denoted as

K_{IC} and called as Fracture toughness[6], then the crack is presumed to have reached the threshold of rapid crack propagation.

This concept can be utilized for obtaining the initial and final crack length for a given loading based on virtual crack extension methods [7].

In the present work special singular crack tip elements are used for both 2-D and 3-D butt and fillet welded geometry for the evaluation of stress intensity factor. For a specified loading, the crack extension lengths and critical crack lengths were obtained by using virtual crack extension methods. The fatigue life assessments of the weld geometry were evaluated by using Paris law.

2. FINITE ELEMENT ANALYSIS OF BUTT& FILLET WELDED JOINT WITH CRACK

Constitutive relation for isotropic materials

The state of stress at a point in a body exhibiting linear elasticity is related to state of strain by Generalized Hooke's law as

$$\{\sigma\} = [C]\{\varepsilon\} \quad (1)$$

Where

$$\{\sigma\} = \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \\ \tau_{XY} \\ \tau_{YZ} \\ \tau_{ZX} \end{Bmatrix}$$ are the six components of stress (2)

and the six components of strain are

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \\ \gamma_{XY} \\ \gamma_{YZ} \\ \gamma_{ZX} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \\ \tau_{XY} \\ \tau_{YZ} \\ \tau_{ZX} \end{Bmatrix}$$

(3)

where E is the Young's modulus of elasticity and G is the shear modulus and ν is the Poisson's ratio.

In compact form

$\{\varepsilon\} = [S]\{\sigma\}$, $[S]$ is the stiffness matrix (4)

$$[S] = [C]^{-1} \quad (5)$$

$$[C] = [S]^{-1} \quad (6)$$

$[C]$ is the elasticity matrix

For plane stress condition the elasticity matrix will be

$$[C] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (7)$$

For plane strain condition the elasticity matrix will be

$$[C] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \quad (8)$$

The nodal displacement is given by $\{a_i\} = \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix}$

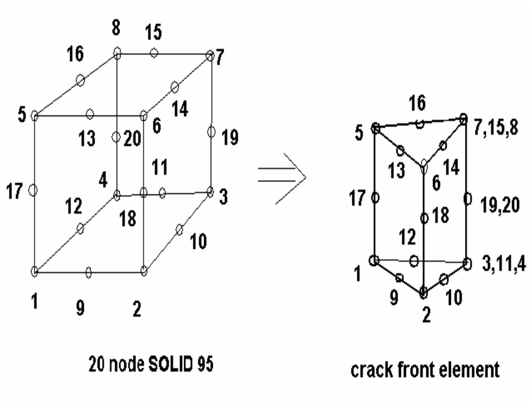

Fig.1: Solid 95 element used in the 3-D analysis

The shape functions for the crack front element are

$$N_1 = \frac{1}{2}\{L_1(2L_1 - 1)(1-r) - L_1(1-r^2)\}$$
$$N_2 = \frac{1}{2}\{L_2(2L_2 - 1)(1-r) - L_2(1-r^2)\}$$
$$N_3 = \frac{1}{2}\{L_3(2L_3 - 1)(1-r) - L_3(1-r^2)\}$$
$$N_5 = \frac{1}{2}\{L_1(2L_1 - 1)(1+r) - L_1(1-r^2)\}$$
$$N_6 = \frac{1}{2}\{L_2(2L_2 - 1)(1+r) - L_2(1-r^2)\}$$
$$N_7 = \frac{1}{2}\{L_3(2L_3 - 1)(1+r) - L_3(1-r^2)\}$$

(9)

$$N_9 = 2\{L_1 L_2(1-r)\}$$
$$N_{10} = 2\{L_2 L_3(1-r)\}$$
$$N_{12} = 2\{L_1 L_3(1-r)\}$$
$$N_{13} = 2\{L_1 L_2(1+r)\}$$
$$N_{14} = 2\{L_2 L_3(1+r)\}$$
$$N_{16} = 2\{L_1 L_3(1+r)\}$$
$$N_{17} = L_1(1-r^2)$$
$$N_{18} = L_2(1-r^2)$$
$$N_{19} = L_3(1-r^2)$$

(10)

L_1, L_2, L_3 and r are the natural co-ordinates, r varies from +1 to –1.

The displacement at any point inside the element is given by

$$u = \sum_{1}^{15} N_i(x,y)u_i$$
$$v = \sum_{1}^{15} N_i(x,y)v_i \qquad (11)$$
$$w = \sum_{1}^{15} N_i(x,y)w_i$$

The strain components are given by

$$\varepsilon_x = \frac{\partial u}{\partial x} = \sum_{1}^{15} \frac{\partial N_i}{\partial x} u_i$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = \sum_{1}^{15} \frac{\partial N_i}{\partial y} v_i \qquad (12)$$
$$\varepsilon_z = \frac{\partial w}{\partial z} = \sum_{1}^{15} \frac{\partial N_i}{\partial z} w_i$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \sum_{1}^{15} \frac{\partial N_i}{\partial y} u_i + \sum_{1}^{15} \frac{\partial N_i}{\partial x} v_i$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \sum_{1}^{15} \frac{\partial N_i}{\partial z} v_i + \sum_{1}^{15} \frac{\partial N_i}{\partial y} w_i$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \sum_{1}^{15} \frac{\partial N_i}{\partial x} w_i + \sum_{1}^{15} \frac{\partial N_i}{\partial z} u_i$$

In matrix form it can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \sum_{1}^{15} \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} \qquad (13)$$

In compact form it can be written as $\{\varepsilon\} = [B]\{a^e\}$,

$$[B_i]=\begin{bmatrix}\frac{\partial N_i}{\partial x} & 0 & 0\\ 0 & \frac{\partial N_i}{\partial y} & 0\\ 0 & 0 & \frac{\partial N_i}{\partial z}\\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0\\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y}\\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x}\end{bmatrix} \quad (14)$$

Hence the $[B]$, the strain nodal matrix can be completely written as, $[B]=[[B_1]....[B_{15}]]$

$$[K^e]=\iiint_V [B]^T[D][B]dV \quad (15)$$

Then by the principle of virtual work $\iiint_V \{\delta\varepsilon\}^T dV-\{\delta a^e\}^T\{q^e\}=0$, where $\{q^e\}$ is the vector of nodal forces,

$\{q^e\}=[K^e]\{a^e\}$, where $[K^e]=[B]^T[D][B]tA$,

Where t is the thickness of the element and A is the area of the element.

By assembling all the element stiffness matrixes the Global stiffness matrix will be obtained. Thus the finite element equations can be finally written as global equations,

$[K]\{\delta\}=\{r\}$, where $[K]$ is global stiffness matrix, $\{\delta\}$ is the vector of nodal displacements and $\{r\}$ is the load vector.

3. STRESS INTENSITY FACTOR & CRITICAL STRESS INTENSITY FACTOR:

Irwin who defined the new variable **K** as the stress intensity factor is to measure the severity of stress in front of the crack tip. For Mode I (Opening Mode)

$$K_I=\sigma(\pi a)^{\frac{1}{2}} \quad (16)$$

The stress and displacement equations can be written in terms of stress intensity factor as

$$\sigma_{xx}=\frac{K_I}{(2\pi r)^{\frac{1}{2}}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \quad (17)$$

$$\sigma_{YY}=\frac{K_I}{(2\pi r)^{\frac{1}{2}}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \quad (18)$$

$$\sigma_{XY}=\frac{K_I}{(2\pi r)^{\frac{1}{2}}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (19)$$

$$u_X=\frac{K_I}{\mu}\left(\frac{r}{2\pi}\right)^{\frac{1}{2}}\cos\frac{\theta}{2}\left[1-2\nu+\sin^2\frac{\theta}{2}\right] \quad (20)$$

$$u_Y=\frac{K_I}{\mu}\left(\frac{r}{2\pi}\right)^{\frac{1}{2}}\sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right] \quad (21)$$

For Mode II (Sliding Mode) in plane strain and far field stress is τ with

$$K_{II}=\tau_{YX}(\pi a)^{\frac{1}{2}} \quad (22)$$

The stress and displacement equations can be written in terms of stress intensity factor as

$$\sigma_{XX}=-\frac{K_{II}}{(2\pi r)^{\frac{1}{2}}}\sin\frac{\theta}{2}\left[2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right] \quad (23)$$

$$\sigma_{YY}=\frac{K_{II}}{(2\pi r)^{\frac{1}{2}}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (24)$$

$$\sigma_{XY}=\frac{K_{II}}{(2\pi r)^{\frac{1}{2}}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \quad (25)$$

$$u_X=\frac{K_{II}}{\mu}\left(\frac{r}{2\pi}\right)^{\frac{1}{2}}\sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right] \quad (26)$$

$$u_Y=\frac{K_{II}}{\mu}\left(\frac{r}{2\pi}\right)^{\frac{1}{2}}\cos\frac{\theta}{2}\left[-1+2\nu+\sin^2\frac{\theta}{2}\right],$$

and $u_Z=0$ (27)

For Mode III,(Tearing Mode) the stress intensity factor can be written as $K_{III}=\tau_{YZ}(\pi a)^{\frac{1}{2}}$ (28)

The stress and displacement field near crack tip in terms of stress intensity factor can be written as

$$\sigma_{XZ}=-\frac{K_{III}}{(2\pi r)^{\frac{1}{2}}}\sin\frac{\theta}{2} \quad (29)$$

$$\sigma_{YZ}=\frac{K_{III}}{(2\pi r)^{\frac{1}{2}}}\cos\frac{\theta}{2} \quad (30)$$

$$\sigma_{XX}=\sigma_{YY}=\sigma_{ZZ}=\sigma_{XY}=0 \quad (31)$$

$$u_X=u_Y=0,\ u_Z=\frac{K_{III}}{\mu}\left(\frac{2r}{\pi}\right)^{\frac{1}{2}}\sin\frac{\theta}{2} \quad (32)$$

If it is required to predict whether the crack will grow or not, then study of stress intensity factor (SIF) is necessary. When a load is applied on a component, the stresses are higher in the vicinity of crack tip which are characterized by the parameter SIF. If SIF of a crack approaches or exceeds an upper limit is known as critical stress intensity factor which is a material property and is usually denoted by K_{IC} for mode I. Stress intensity factor is a parameter to measure severity of stress at the crack tip but critical stress intensity factor is the limit on SIF such that if SIF exceeds beyond the critical stress intensity factor, the crack becomes unstable. If the plate thickness is significantly greater than the size of plastic zone the condition of plane strain exists. Critical stress intensity factor for thin plates depends on plate thickness and it's value is rarely provided as function of thickness in literature. The critical SIF is greater than the corresponding value in plane strain, so if a designer finds that a component is subjected to plane stress but the critical value of plane strain is available, then that value can be used safely for the design.

MATERIAL	YIELD STRESS, *Mpa*	$K_{IC}, Mpa\sqrt{m}$
Mild steel	240	220
Medium carbon steel	260	54
Alloy steel	860	99
Stain less steel	2240	80-150

When $K_I \geq K_{IC}$, there will be unstable crack growth

4. SPECIMEN GEOMETRY & MATERIAL PROPERTY:

The value of Young's Modulus of elasticity is E= $2*10^5$ Mpa and Poission's ratio ν =0.3

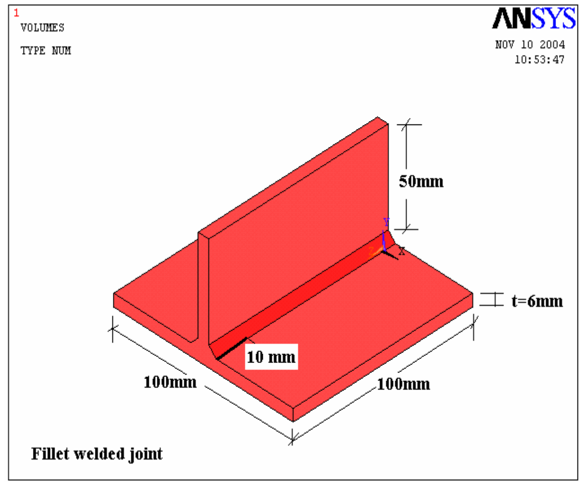

Fig. 2: Fillet welded specimen with initial crack of 10mm

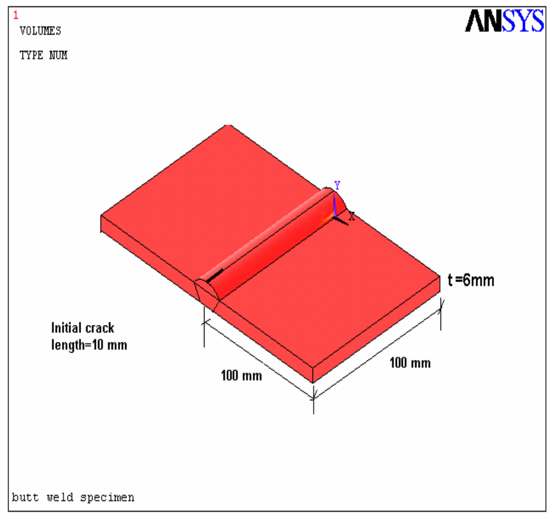

Fig. 3: Butt welded specimen with initial crack of 10mm

5. RESULTS & DISCUSSIONS:

5.1 Meshing and Boundary Condition of Butt-welded Cracked joint

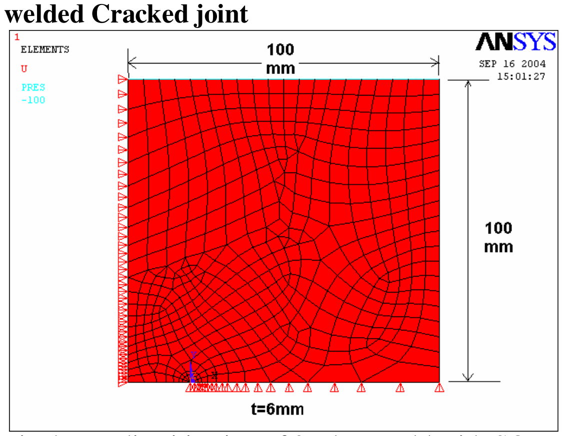

Fig 4 : FE discritization of 3D butt weld with SOLID 95 using crack tip elements

The 3D model has been given two types of fatigue loading and the stress intensity factor, strain energy release rate has compared with the theoretical value. At each load step a virtual extension has been given to the existing crack length and the stress intensity factor is calculated.

The virtual extension of crack continues till the stress intensity factor approach the critical stress intensity factor. The critical stress intensity factor is a material property and has a value of $54.0\, Mpa\sqrt{m}$ for medium carbon steel

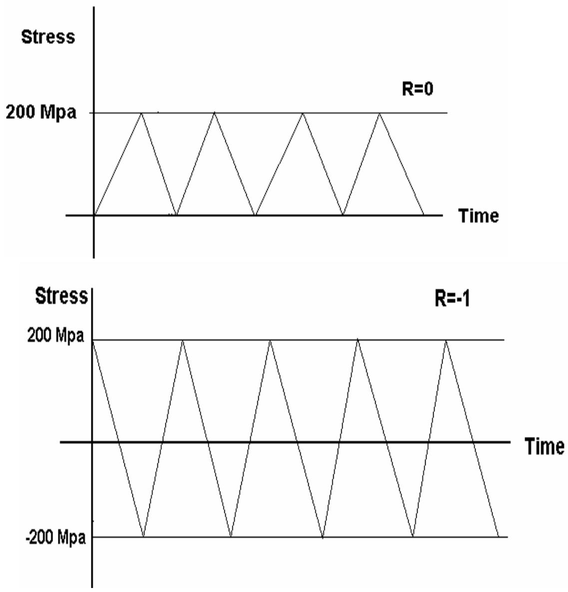

Fig. 5 : Fatigue Loading applied 3D butt & fillet weld specimen

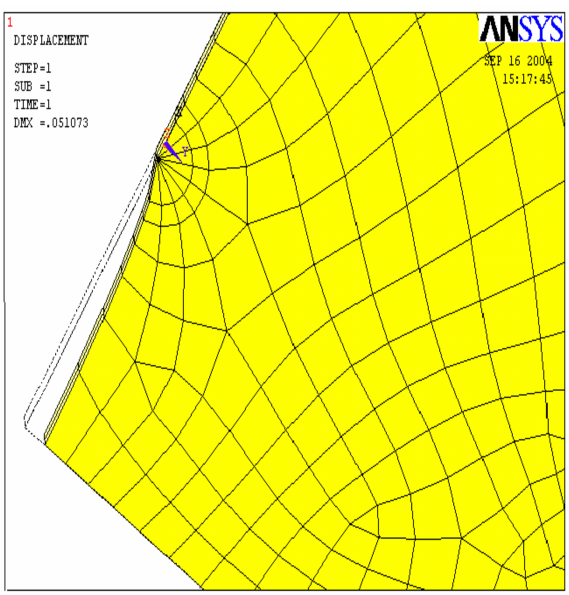

Fig 6: Deformed shape of joint

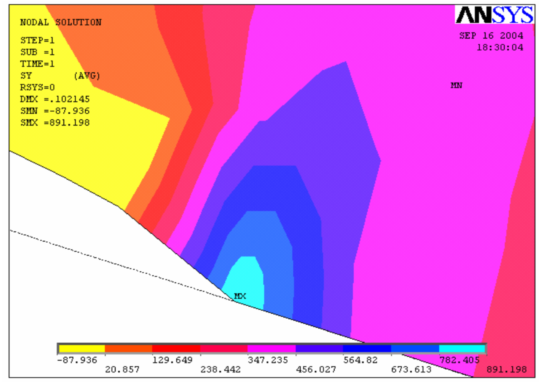

Fig 7: Stress distribution near crack front

Table-1

Computed values of G_I and K_I for 3D butt welded joint with crack

Crack length (mm)	K_I (SIF) (Mpa√m) by (Fracture Mechanics)	K_I (SIF) (Mpa √m) by (FEM)	G_I (SERR) (KJ/m^2) by (Fracture Mechanics)	G_I (SERR) (KJ/m^2) by (FEM)
10	35.777	33.72	5.832	5.8
12	39.192	37.155	6.99	7.07
14	42.33	40.41	8.165	8.393
15	43.8	42.179	8.748	9.165
16	45.195	43.784	9.307	9.814
18	48.1	46.519	10.42	11.21
19	49.6	48.114	11.28	11.90
20	51.04	49.51	11.95	12.633
21	52.35	51.113	12.89	13.42
22	53.59	52.5	13.287	13.85
23	54.8	54.09	13.481	13.98

From the result it has been concluded that at crack length of 23mm the stress intensity factor by FEM approach the critical stress intensity factor, so 23mm is the critical crack length for the above model.

At crack length of 23 mm, $K_I \geq K_{IC}$, so 23mm is the critical crack length beyond which unstable propagation of crack occurs.

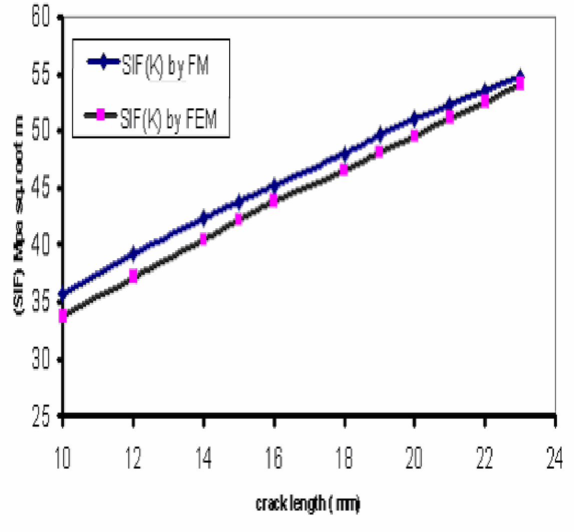

Fig 8: SIF for varying crack lengths

The stress intensity factor increases with increase in crack length.

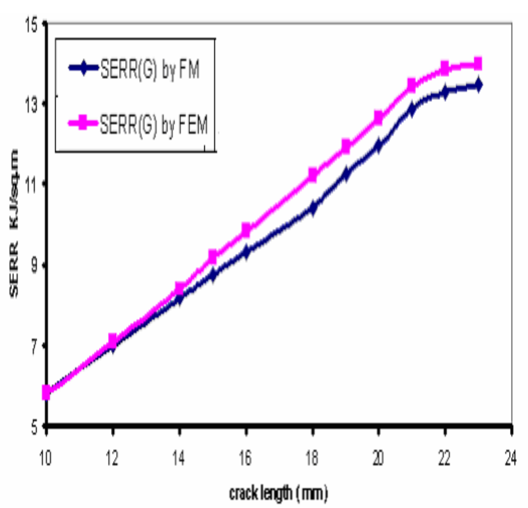

Fig 9: SERR for varying crack lengths

The SERR increase with increase in crack length and attains a constant value near to the critical crack length.

5.3 Calculation of Fatigue life for Butt Welded Joint

Paris Law for fatigue life prediction:

$$N_p = \frac{a_o^{\left(\frac{-m}{2}+1\right)} - a_f^{\left(\frac{-m}{2}+1\right)}}{\left(\frac{m}{2}-1\right) C f^m \left(\frac{a_o}{W}\right) (\Delta\sigma)^m \pi^{\frac{m}{2}}}$$

Where N_p is the number of cycles required to propagate the crack from initial crack length a_o to final crack length a_f.

C And m is the material constants.

$f\left(\frac{a_o}{W}\right)$ is the configuration factor.

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

The value obtained is **21700** number of cycles for R= 0 (Repeated cycles) &
2724 number of cycles for R= -1 (Completely reversed cycle)

5.4 Meshing and Boundary Condition of Fillet-welded Cracked joint

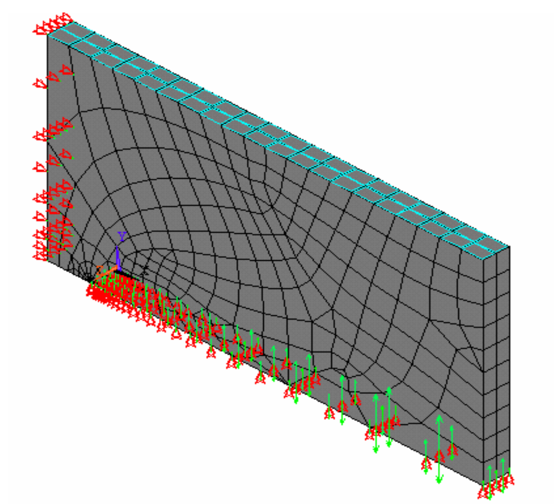

Fig. 10: FE meshing and loading of fillet welded joint with crack

All nodes on bottom face are constrained to Y-displacement and all all nodes of left side face are constrained to X-displacement.

Table-2

Computed values of K_I, and G_I for 3D fillet weld joint with crack

Crack Length (mm)	SIF (K_I) Mpa√m (FM)	K_I (SIF) (Mpa√m) (FEM)	G_I (SERR) (KJ/m^2) (FM)	G_I (SERR) (KJ/m^2) (FEM)
10	35.8	34.97	6.12	6.22
11	37.54	36.998	6.89	6.97
12	39.217	38.778	7.65	7.715
13	40.81	40.81	8.34	8.48
14	42.611	42.81	9.25	9.357
15	44.128	44.81	10.6	10.3
16	45.6	46.84	10.98	11.227
17	47.00	48.88	11.65	12.186
18	48.41	51.35	12.96	13.437
19	51.48	53.5	13.68	14.55
20	53.8	55.54	14.98	15.7

From the result it is concluded that at crack length of 20mm the stress intensity factor by FEM approach the critical stress intensity factor, so 20mm is the critical crack length for the above model.

At crack length of 20 mm $K_I \geq K_{IC}$, so 20mm is the critical crack length beyond which unstable propagation of crack occurs.

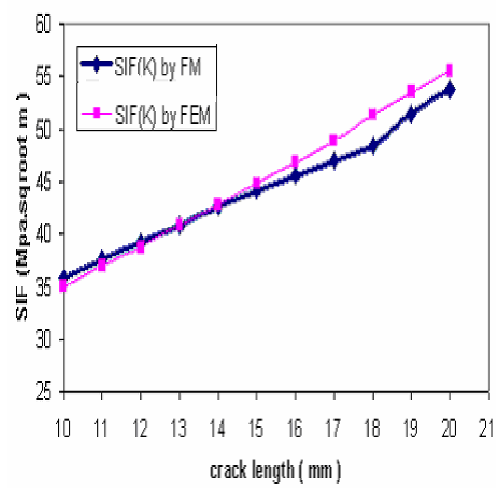

Fig 11: Variation of SIF with varying crack length.

The stress intensity factor increase with increase in crack length.

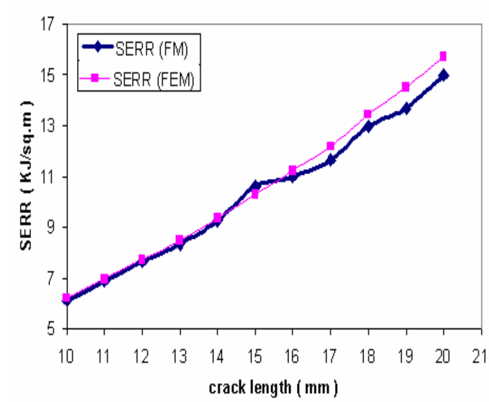

Fig 12: Variation of crack length verses SERR

The SERR increase with increase in crack length

5.5 Calculation of Fatigue Life for 6mm Fillet Welded Joint

Paris Law for fatigue life prediction:

$$N_p = \frac{a_o^{\left(\frac{-m}{2}+1\right)} - a_f^{\left(\frac{-m}{2}+1\right)}}{\left(\frac{m}{2}-1\right) C f^m \left(\frac{a_o}{W}\right) (\Delta\sigma)^m \pi^{\frac{m}{2}}}$$

Where N_p is the number of cycles required to propagate the crack from initial crack length a_o to final crack length a_f.

C And m is the material constants.

$f\left(\frac{a_o}{W}\right)$ is the configuration factor.

$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$

The value obtained is **18700** number of cycles for R= 0 (Repeated cycle) &

2362 number of cycles for R= -1 (Completely reversed cycle).

Here the initial crack length was 10mm and at crack length of 20 mm, it has been found that the stress intensity factor becomes greater than the critical stress intensity factor whose value is 54.0 $Mpa\sqrt{m}$

6. CONCLUSIONS:

A constant amplitude variable loading of stress ratio(R) equal to 0 & -1 have been applied to the entire butt and fillet welded models. At crack lengths of 23mm, and 20mm the stress intensity factor (SIF) approach the critical stress intensity factor, so theses are the critical crack lengths for the above 3D models of butt and fillet welded joints. The crack will propagate rapidly when this critical crack length is reached. For butt-welded joint the number of cycles the specimen can withstand before failure is 21700 for repeated cycle(R= 0) and 2724 for completely reversed cycle (R= -1), the number of cycles the specimen can withstand before failure for fillet-welded joint is 18700 for (R= 0) and 2362 for (R= -1).

- The stress intensity factor (SIF) K_I and strain energy release rate (SERR) G_I increases with the increase in crack length of butt-welded and fillet welded joint
- The stress at the crack tip and mid-side node displacement increases with increase in crack length for both butt and fillet welded joints with cracks.
- Plastic zone size evaluated by FE method matches well with the available literature.
- The SERR and SIF values obtained by FEM are match well with the literature.
- Fatigue life of the specimen is predicted based on Paris law and the critical crack length has been obtained by FEM.

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